# Interference of quantum channels in single photon interferometer

Guo-Yong Xiang, Jian Li, and Guang-Can Guo
*Key Laboratory of Quantum Information, University of Science and Technology of China, CAS, Hefei 230026, People's Republic of China*

We experimently demonstrate the interference of dephasing quantum channel using single photon Mach-Zender interferometer. We extract the information inaccessible to the technology of quantum tomography. Further, We introduce the application of our results in quantum key distribution.

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Macroscopic quantum systems can never be isolated from their environments. It leads to decoherence which destroys superpositions. And when a qubit transmits through a quantum channel, the interaction between qubit and quantum channel is inevitable. The decoherence in quantum channel affects the distance and quantity of quantum information transmitting. So it is important to know what happen when quantum information transmit through noisy quantum channel. The technology of quantum process tomography[1, 2] can be used to character the quantum channels.

But J. Aberg[3] find that we can not specify the action of the sinultaneous operation of both maps although we known the individual quantum channels. It is said that when a superposition state pass through two quantum channels, we can not know the information of output state exactly by using the technology of quantum process tomography. Single particle interference can help us extract information inaccessible to conventional process tomography. D. K. L. Oi have given a measure of coherent fidelity, the maximum interference visibility, and the closest unitary operator to a given physical process under this measure[4].

Here, We give an interference visibility of two quantum processes which have same environment degree and carry out an experiment to demonstrate it. The environment qubit is the time qubit from birefrigence of quartz crystal in the experiment, i.e. quantum channels we used is the dephasing channel. We find that there are plentiful information of inteference which is the information inaccessible to conventional process tomography[1, 2].

When a single qubit state transmits through two quantum channels (Fig. 1), how can we known the output state? The technology of quantum tomography can obtain the densities of output states in each paths. But the whole density of the output state can not be fixed, i.e. there is other information which have not been extracted. D. K. L. Oi[4] shows that single particle Mach-Zender inereference can help us. Different visibilities show quantum information not presented in the two individual quantum channel. When the different environment degree ($E$ and $F$) appended to the operations of the upper and lower arms, D. K. L. Oi presents the interference patterns as

$$Tr[u_0^+ v_0 \rho], \tag{1}$$

where $\rho$ is the input state, $u_0$ and $v_0$ are the first Kraus operators for the quantum processes $U$ and $V$ in upper and lower arms. If the input state is the maximally mixed state, the interference pattern depends on $\frac{1}{d}Tr[u_0^+ v_0]$.

In Eq. 1, The interference patterns only depend on the first Kraus operators $u_0$ and $v_0$. But it find that when the environment degree is same to the operations of the both arms (Fig. 2), the interference pattern will depend on the four Kraus operators $u_i$ and $v_i$. The beamsplitters in Fig. 2 and the phase shifter are modeled by the unitary operators $U_b$ and $U_p$ respectively,

$$U_b = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}, U_p = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}. \tag{2}$$

The original state $\rho_{in} = |0\rangle\langle 0| \otimes \rho$ of the system on internal Hilbert space and the two-dimensional Hilbert space of path degree is evolved as

$$\rho_{in} \mapsto U_b(|0\rangle\langle 0| U + |1\rangle\langle 1| V)U_p U_b \rho_{in} U_b^\dagger U_p^\dagger (|0\rangle\langle 0| U^\dagger + |1\rangle\langle 1| V^\dagger) U_b^\dagger \tag{3}$$

where $|1\rangle$ and $|0\rangle$ represent the upper and lower path. The probability of finding the particle in the horizontal direction, i.e. in the $|0\rangle$ state, is

$$P_{|0\rangle}(\phi) = \frac{1}{2}(1 + Ree^{i\phi} Tr[U^+ V \rho \otimes |e_0\rangle\langle e_0|]). \tag{4}$$

So $P_{|0\rangle}(\phi)$ is decided by

$$Tr[U^+ V \rho \otimes |e_0\rangle\langle e_0|] = \sum_i Tr[u_i^+ v_i \rho], \tag{5}$$

where

$$\{u_i\} = \{\langle e_i| U |e_0\rangle\}, \{v_j\} = \{\langle e_j| V |e_0\rangle\} \tag{6}$$

are the Kraus operators of $U$ and $V$. Where $\{|e_i\rangle\}$ and $\{|e_j\rangle\}$ are the orthornomal bases of E, $|e_0\rangle$ is the initial state of E. Specially, If the input state is the maximally mixed state, the interference pattern depends on

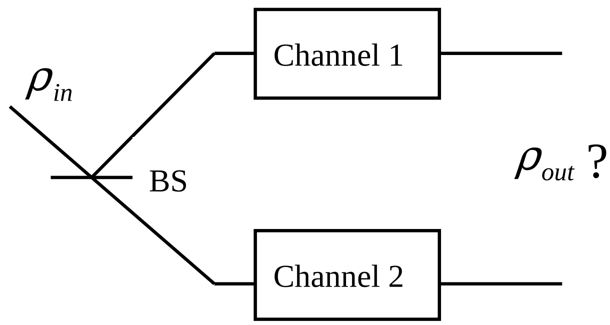


FIG. 1: After the beam splitter(BS), the input state $\rho$ will transmit through a coherent superposition of quantum channel 1 and quantum channel 2. The output state $\rho_{out}$ can not be determined by conventional process tomography

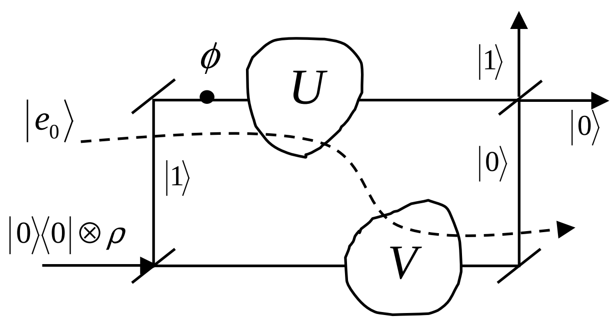


FIG. 2: Interference of two quantum channels. The environment degree of the two channels is same. After the second BS, the environment qubit will be traced out

$\frac{1}{d}Tr[u_i^+ v_i]$, which is the interference patterns of quantum channels.

So the interference patterns determined by all four kraus operators of $U$ and $V$ and their relative phase. It will give richer interference pattern than that Eq. 1 gives us.

The visibility of the intereference pattern is the effects of the indistinguishability of the two paths that the particle transmitted through. According to the intereference pattern and visibility, it can be determined whether the two quantum processes are identical or different (see [5] for related problem). Because of the birefringence of ordinary light ($o$ light) and extra-ordinary light ($e$ light) in BBO crystal, we choose the time degrees of freedom of photon passing through a BBO crystal to be the environment qubit ($|t_o\rangle$ to for ordinary light and $|t_e\rangle$ for extraordinary light). There are two birefrigence crystals in the upper and lower arms of Mach-Zender interferometer respectively (see Fig. 3). The Kraus operators of two arms can be represented by

$$\begin{aligned} \{u_m\} &= \{\langle a_i|b_j\rangle |a_i\rangle \langle b_j|\}, \\ \{v_n\} &= \{\langle a_i'|b_j'\rangle |a_i'\rangle \langle b_j'|\} \end{aligned} \tag{7}$$

Where $m, n = 0, 1, 2, 3$, and $a_i, b_j, a_i', b_j'$ are the angles of the fast axis of the crystals relative to horizontal direction; $i, j = o, e$, and $\{|a_o\rangle = \cos a\,|H\rangle + \sin a\,|V\rangle, |a_e\rangle = -\sin a\,|H\rangle + \cos a\,|V\rangle\}, \{|b_o\rangle = \cos b\,|H\rangle + \sin b\,|V\rangle, |b_e\rangle = -\sin b\,|H\rangle + \cos b\,|V\rangle\}$ are orthogonal basis respectively. Here, $m, n$ are defined by the sequence of the four pulse following the second quartz crystal in the two arms respectively. For example, $u_0 = \langle a_0|b_0\rangle |a_0\rangle \langle b_0|, u_1 = \langle a_0|b_e\rangle |a_0\rangle \langle b_e|, u_2 = \langle a_e|b_0\rangle |a_e\rangle \langle b_0|, u_3 = \langle a_e|b_e\rangle |a_e\rangle \langle b_e|$.

According to Eq. 4, the interference patterns are determined by

$$ve^{i\phi} = \sum_i Tr[u_m^+ v_n \rho]. \tag{8}$$

The experimental setup is represented in Fig. 3. A

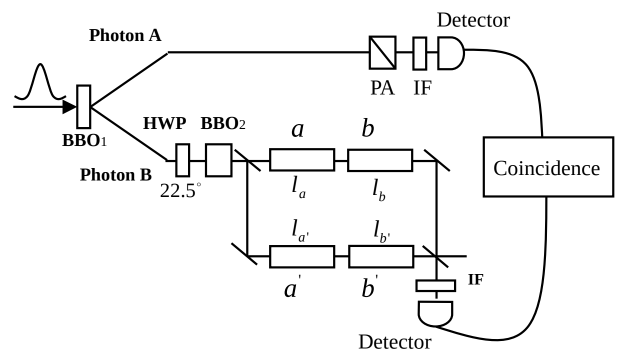


FIG. 3: Experimental setup for the interference of quantum channels. PA represents polarization analyzer and IF represents interference filter; HWP are half waveplates.

pulse of ultraviolet (UV) light pass through a BBO crystal ($1.0mm$, cut for type-I phase match) . The UV pulse is frequency-doubled pulse (less than $200fs$ with $82MHz$ repetition and $390nm$ center-wavelength) from a mode-locked Ti: sapphire laser (Tsunami by Spectra-Physics). Through the SPDC process, photon pairs are generated with $780nm$ center-wavelength. By detecting one photon of the pairs (with single photon detector after a $4nm$ FWHM interference filter at $780nm$), the other one (photon 1) can be prepared into any polarization state[6, 7] to be sent into Mach-Zender interferometer.

After a half-wave plate fixed $22.5^0$ and a $5.0mm$ thick BBO crystal (After which, the separation of wavepackets between H(o)- and V(e)-polarized light is about $580\mu m$) and Because the coherent length of the wavepacket is about $150\mu m$ ($4nm$ FWHM interference filter is inserted before each detector), Photon 1 is prepared in the maximally mixed state. Then it sent into Mach-Zender interferometer (Fig. 3). There are two quarz crystals in the upper and lower arms respectively. The two short ones ($l_1$) separate H(o)- and V(e)-polarized light $190\lambda$ (about $150\mu m$), and the two longer ones ($l_2$) separate H(o)- and V(e)-polarized light $398\lambda$ (about $310\mu m$). The angles of their optical axes relative to horizontal plane are $a, b, a', b'$ (see Fig. 3). Then beams from the two arms interact at the second beam splitter, and photon 1 is detected by single photon detector below interferometer

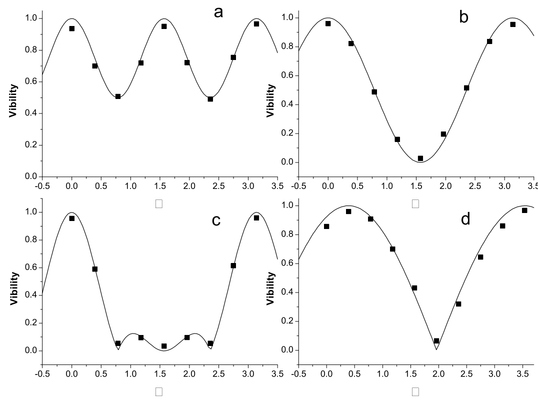


FIG. 4: The experimental results of interference of quantum channel.

after a $4nm$ FWHM interference filter. The signals from two detectors are coincided within a $5ns$ timing window by using a coincidence counter (EG&G, TAC/SCA).

The maps according to quantum channel are changed by adjusting the angles ($a, b, a'$, and $b'$) and the arrangement of four quartz crystals. we will observe the visibility of interference of different quantum channels. 1), we choose $l_a = l_{b'} = l_1$, $l_b = l_{a'} = l_2$, $b = a' = 0$, and $a = b' = \beta$, then the visibility is $v = 1 - \frac{\sin^2(2\beta)}{2}$ which is always more than 50% (Fig. 4a); 2), $l_a = l_{b'} = l_1$, $l_b = l_{a'} = l_2$, $a = a' = 0$, and $b = b' = \beta$, then the visibility is $v = \cos^2\beta$ (Fig. 4b); 3), $l_b = l_{b'} = l_1$, $l_a = l_{a'} = l_2$, $b = a' = 0$, and $a = b' = \beta$, then the visibility is $v = \cos^2\beta\cos(2\beta)$ (Fig. 4c); 4), There is a half-wave plate in the upper and lower arms, *i.e.* $l_a = l_{b'} = l_b = l_{a'} = 0$, and the angles of the one in upper and lower arm are fixed in $\frac{\pi}{8}$and $\beta$ respectively, then the visibility is $v = \left|\cos(\beta - \frac{\pi}{8})\right|$ (Fig. 4d). Because, to the maximally mixed states input, the outpt state are still maximally states after the quartz crystals in both arms and the fidelity of the output states of both arms are always 100% which are independent of the maps in the arms[7], the change of the visibilities according to $\beta$ is the information inaccessible to conventional process tomography.

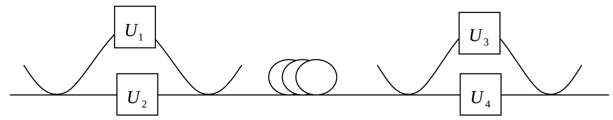


FIG. 5: The schematic representation of QKD scheme based on unbalanced fiber Mach–Zender interferometers.

Our results can be used to explain the low visibility of Mach–Zender interferometer in QKD system([8, 9, 10, 11]). A typical fiber QKD scheme is based on unbalanced fiber Mach–Zender interferometers (Fig. 5). Here we suppose the common quantum channel between two unbalanced Mach–Zender interferometers was identity. So this QKD system can be simplified to one Mach–Zender interferometer (see Fig. 3),and $U_i$ ($i = 1, 2, 3, 4$) correspond to four quartz crystals in our experiment. Eq. [5] gives the visibility of any input sate. Our further work will demonstrate the visibility of QKD scheme when the common quantum channel between two unbalanced Mach–Zender interferometers is not identity

In summary, we have demonstrated the interference of quantum channels single photon Mach-Zender interferometer. Our results present the information inaccessible to the techonology of quantum process tomography. This work can lead to further investigation into the phase between operations and structure and geometry of the CP maps.

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